Reply by Authors to R. J. Black

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THIS paper was based on the fundamental concept that the ground force and moment are essentially path dependent rather than time dependent. Path dependence is indicated by the relatively sophisticated efforts of Pacejka[1] and the simplified approaches of Segel[2] and Rogers[3] and experimentally by Pacejka[1] and Clark.[5] Admittedly in the case of aircraft tires, path dependence has not been irrefutably established by coordinated theoretical and experimental programs at actual speeds and frequencies, but there is no indication that their behavior will be fundamentally different from that of automotive tires. This path dependence leads to one of the most serious objections to the extended Moreland tire theory. To illustrate this point we repeat here Eq. (A2) and (A3) from the appendix of the original paper

$$M = k_t\alpha_1$$
$$C_yF = -(\alpha_1 + C\dot{\alpha}_1)$$

These equations state that the force and moment response depend solely on the slip angle, regardless of the path. As a consequence, the tire response, for equal slip angles, is the same whether the tire is undergoing pure yaw or pure sideslip. This is in fundamental disagreement with the theoretical predictions of Segel,[2] Saito,[6] and Pacejka,[1] as well as with the recent experimental measurements by Clark.[5] These works show that the response due to pure sideslip and pure yaw, for equal slip angles, are distinctly different, except for the quasi-steady-state condition where $\Omega = 0$.

Another key concept is the question of what evidence constitutes rigorous experimental verification that the ground force and torque on the tire are represented accurately. The motion is described by a swivel angle and a lateral displacement of the wheel rim plane; in general, these are separated by a phase angle. The ground force will be made up of a component consisting of a magnitude and a phase angle due to lateral displacement and another component consisting of a magnitude and phase angle due to swivel angle. These two components must be added vectorially to yield the lateral force. A similar situation holds for the aligning torque. The generalized work of the ground on the wheel is due to the force acting through the appropriate lateral displacement plus the torque acting through the appropriate angular displacement, with all magnitudes and phase angles properly accounted for. If now the tire is made part of a dynamic system (e.g., a shimmy problem or a tire testing machine), it is obvious that the homogeneous solution of the system equations of motion will agree with experiment provided only that the generalized work of the tire is correct. Note that the same generalized work is possible for different combinations of force and torque magnitudes and phase angles. Thus a favorable comparison of theory and experiment for a system response is a necessary, but not sufficient, condition to verify, the tire representation.

The authors agree with R. J. Black that the region of interest for shimmy of aircraft landing gear corresponds to the reduced frequency range $0.1 < \Omega < 2.0$. However, the authors take exception to the claim that the extended Moreland tire theory is accurate over this region. Existing experimental data by Brewer,[7] Ginn,[8] and Pacejka,[4] demonstrate that the dip in the aligning torque amplitude response, under cyclic yaw, generally occurs in the reduced frequency range of $1.0 < \Omega < 2.0$, while recent work by Clark[5] on model aircraft tires shows that it may occur at a reduced frequency as low as 0.7 rad./ft. Thus although these values lie in the region of interest defined above, an examination of Fig. 3 and 8 of the original paper shows that the extended Moreland theory does not agree even qualitatively with the experimental data.

A new aircraft tire dynamometer test facility is nearing completion at Wright-Patterson Air Force Base and will provide all necessary frequency response data under operational conditions. The data acquisition and reduction of the facility will be completely automated. The authors feel that the four frequency response functions will be adequately represented by nine parameters. A rather extensive investigation[9] is being conducted on smaller size aircraft tires using the facility referenced in the original paper.

The inaccuracies in the extended Moreland theory, both in the relationship to the physics and in the lack of qualitative agreement in the torque-yaw frequency response data, and the chronic occurrence of shimmy in service suggests a need for scrutinizing the tire representation. It was to this need that the paper was addressed.

Received November 29, 1971.

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References

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[4] Pacejka, H. B., "Dynamic Frequency Response of Pneumatic Tires to Lateral Motion Inputs," *Second Conference on Motor Vehicles and Motor Engineering*, Oct. 1971, Sopron, Hungary.

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[8] Ginn, J. L. et. al., "The B. F. Goodrich Tire Dynamics Machine," *SAE Journal*, Vol. 70, July 1962, p. 88.

[9] Air Force Contract No. F33615-72-C-1268, "Frequency Response Measurements of Yawed Aircraft Tires," B. F. Goodrich Co.

Errata

Errata: "Parachute Critical Opening Velocity"

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[*J. Aircraft* **8**, 675–676 (1971)]

A DECIMAL error was reported by G. W. H. Stevens in a private communication to me in November. The error was found in the parameter $\gamma_{avg}/\rho D_0$ computed for the 28-ft parachute data. Thus, the data correlation shown in this Note is not valid.

Received December 13, 1971.